EFFECT OF STEEPNESS OF RISE AND FALL OF THE INPUT PULSE ON THE RESPONSE OF PULSE AMPLIFIERS (PART I)*

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ABSTRACT. The effect of steepness of pulse-fronts on the response characteristics of RC-coupled pulse amplifiers has been studied. The analytical expressions of the responses of the amplifier to pulses of the following types have been derived : (*i*) Ramp function input pulse. (*ii*) A pulse with linear rise and fall and (*iii*) a saw-tooth pulse. In the case of a ramp function input pulse expressions have been deduced relating the rise and delay times of the reproduced pulse with the rise time of the input pulse. In the cases of pulses having sharp rise and fall, formulae have been derived for the maximum output voltage obtainable and the time corresponding to this maximum, as a function of times of rise and fall of the input pulse. The response characteristics for all the interesting cases have been plotted.

INTRODUCTION

Pulse amplifiers are used not only in various branches of electronics but also in almost every work of nuclear physics where it is necessary to amplify the signal coming out of a detector, *e.g.*, the ionisation chamber, proportional counters, etc. In various types of work, specially in experimental problems of nuclear physics, pulse amplifiers are meant only to reproduce the leading edge of the incoming pulse faithfully. The leading edge of the pulse is generally assumed to rise in a very short time and it is desired that the rise time and delay time of the pulse amplifier be very small.

Since all this information about the pulse amplifier is desirable before the start of the actual experiment, the response characteristics of the pulse amplifier to a step-function input pulse (figure 1) which rises from its initial to final value instantaneously, have been obtained theoretically and are available in standard text-books on pulse amplifiers (Valley and Wallman, 1948). An actual pulse can never have such an abrupt rise. The pulse from an ionisation chamber or other types of detector in nuclear physics requires a finite build-up time. So the response of a pulse amplifier calculated by assuming the input voltage to be a step function, does not always demonstrate physically accurate pictures.

To get a correct idea about the output of the pulse amplifier, it is necessary for the purpose of theoretical computations to assume such an

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input pulse that is a very good approximation to the actual pulse. A sharply rising pulse front may be well represented by a linearly rising pulse in practically all the cases. So the input pulse can be assumed to be one that rises linearly to its final value and then flattens out (figure 2). Such a type of pulse is called a ramp function. It is also easy to express this ramp function pulse analytically as given below :

$$e(t)=K\left[\frac{t}{t_1}\,u(t)-\frac{(t-t_1)}{t_1}\,.\;u(t-t_1)\right] \quad \dots \quad (1)$$

where K represents the height of the pulse and $u(t)$ and $u(t-t_1)$ are unit step functions beginning at times $t=0$ and $t=t_1$ respectively.

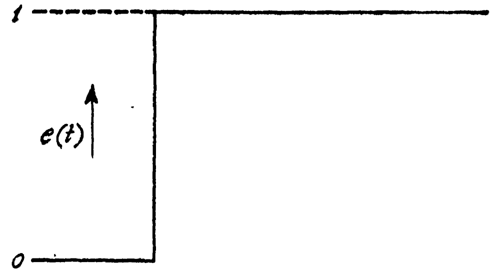

FIG. 1 A step function input pulse.

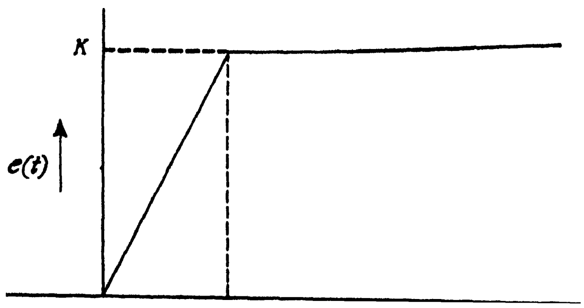

FIG. 2 A ramp function input pulse.

In many cases the incoming pulses are not flattened at the top but begin to fall sharply after reaching the final amplitude. The fall can be taken to be linear in many cases (figure 3). Even when the fall is exponential, the initial portion of the decay is practically linear in most of the cases. This pulse can be represented by the expression :

$$e(t)=K\left[\frac{t}{t_1}\,u(t)-\frac{t_2}{t_1}.\;\frac{(t-t_1)}{(t_2-t_1)}u(t-t_1)+\frac{(t-t_2)}{(t_2-t_1)}.\;u(t-t_2)\right] \quad \dots \quad (2)$$

When $t_1 = t_2$, the pulse shown is figure 3 takes the form of a saw-tooth pulse (figure 4). This pulse is expressed by the following analytical relation :

$$e(t) = K\frac{t}{t_1}[u(t) - u(t-t_1)] \tag{3}$$

The subject of study in this paper is to determine the response functions of typical RC-coupled pulse amplifiers (figure 5) to these types of commonly-occurring pulses. Attention has been concentrated mainly on the effect of variation of steepness in the case of ramp function input pulse on the shape of the output voltage and the rise time and delay time of reproduced pulse. When the pulse does not flatten out at the top (figures 3 and 4), main consideration is given to the determination of the maximum output voltage obtainable, the time at which this maximum occurs and the faithfulness with which the input pulse is reproduced.

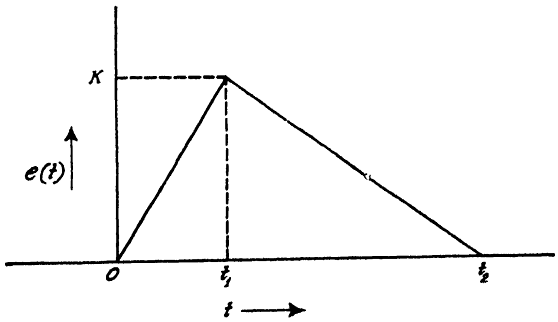

Fig. 3. A pulse with linear rise and fall.

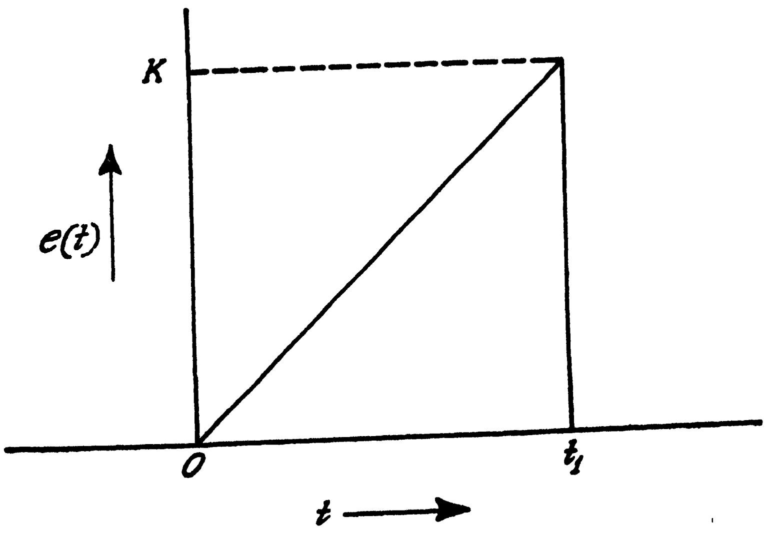

Fig 4. A saw-tooth pulse.

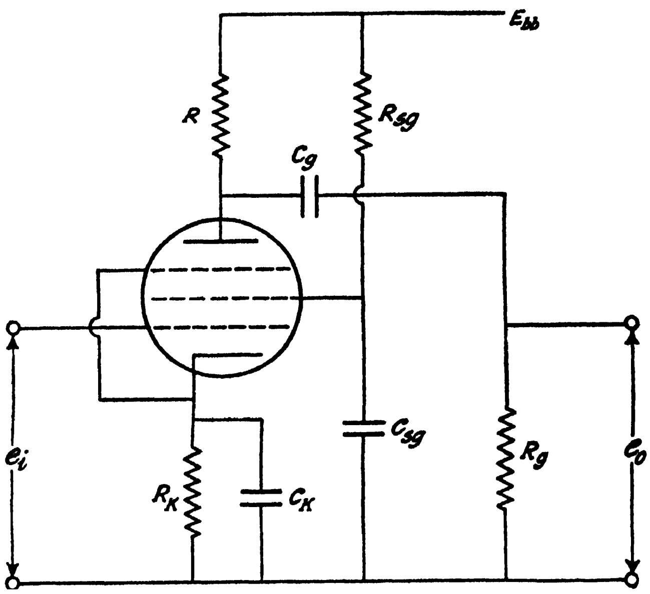

Fig. 5. An RC-coupled pulse amplifier.

RESPONSE TO A RAMP FUNCTION INPUT PULSE

We shall assume that only the high frequency equivalent circuit requires consideration in determining the reproduced output of the leading edge.

In the following analysis we shall always denote the Laplace transform of the voltage $e(t)$ by $e(p)$.

The high frequency equivalent circuit of the RC-coupled pulse amplifier is shown in figure 6, where R is the load resistance, C is the combination of stray and wiring capacitances and g_m denotes the mutual transconductance of the vacuum tube.

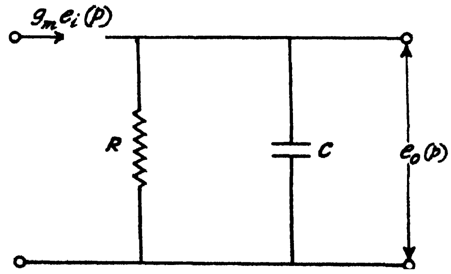

Fig. 6. High frequency equivalent circuit of the amplifier.

If $e_0(p)$ is the output voltage of the amplifier, we have

$$e_0(p)=g_m e_i(p).\ \frac{R}{1+pCR} \quad \ldots \quad (4)$$

From equation (1) we obtain the Laplace transform of the input voltage

$$e_i(p) = K\left[\frac{1}{p^2 t_1} - \frac{\epsilon^{-pt_1}}{p^2 t_1}\right] \qquad \ldots \quad (5)$$

If the resulting equation obtained by putting (5) into (4) be normalized with the substitution $t = t/RC$ and $e_0(t) = \dfrac{e_0(t)}{g_m KR}$, we have

$$e_0(p) = \frac{1}{t_r} \cdot \frac{(1 - \epsilon^{-pt_r})}{(1+p)\, p^2} \qquad (6)$$

where

$$t_r = t_1/RC$$

The inverse Laplace transform of (6) gives

$$e_0(t) = \frac{1}{t_r}(\epsilon^{-t} + t - 1), \quad 0 < t \leq t_r$$

and (7)

$$e_0(t) = \frac{1}{t_r}\left[\epsilon^{-t}(1 - \epsilon^{t_r}) + t_r\right], \quad t \geq t_r$$

Thus the response of a single stage *RC* amplifier is obtained for the case of a ramp function input pulse.

Now, the problem is to find out the voltage response of a chain of amplifiers, as shown in figure 7, to a ramp function. It will be assumed

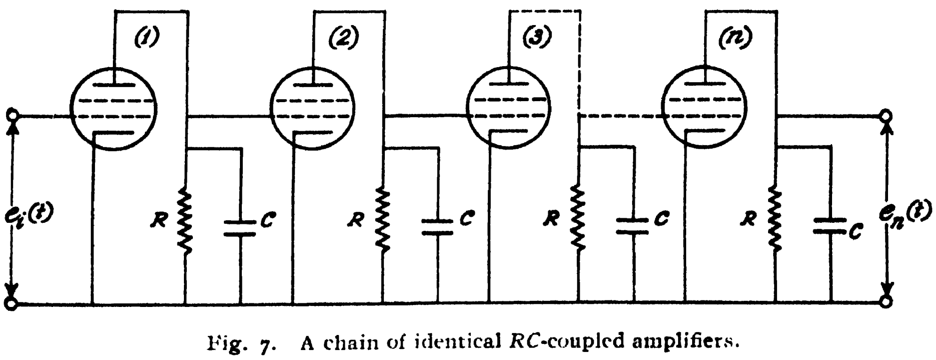

Fig. 7. A chain of identical *RC*-coupled amplifiers.

that all the stages are identical.

It is easy to show that the transform of the voltage developed across the *n*th tube is given by

$$e_n(p) = \frac{(g_m R)^n}{(1 + pCR)^n}\, e_i(p) \qquad \ldots \quad (8)$$

where $e_n(p)$ is the voltage developed across the load resistance of the *n*th tube. Normalizing by the substitution $t = t/RC$ and $e_n(t) = e_n(t)/(g_m R)^n$, we have

$$E_n(p) = \frac{1}{t_r}\left[\frac{1}{p^2(1+p)^n} - \frac{\epsilon^{-pt_r}}{p^2(1+p)^n}\right] \quad \ldots \quad (9)$$

By evaluating the transform of (9), we obtain

$$e_n(t) = \frac{1}{t_r}\left[(t-n) + \frac{1}{(n-1)!}\cdot \epsilon^{-t}\left\{\sum_{r=0}^{(n-1)} {}^{(n-1)}C_r (r+1)!\; t^{(n-r-1)}\right\}\right], \quad 0 \leq t \leq t_r \quad \ldots \quad (10)$$

and

$$e_n(t) = 1 + \frac{1}{t_r(n-1)!}\cdot\left[\left\{\sum_{r}^{(n-1)} {}^{(n-1)}C_r t^{(n-r-1)}(r+1)!\right\} - \epsilon^{t_r}\left\{\sum_{r=0}^{(n-1)} {}^{(n-1)}C_r \cdot (t-t_r)^{n-r-1}\cdot (r+1)!\right\}\right], \quad t \geq t_r \quad \ldots \quad (11)$$

With the help of (10) and (11) we can find the output voltage after any number of stages. Figures 8-11 show the responses of RC-coupled amplifiers to a ramp function input with different rise times, *e.g.*, (*i*) 0.5, (*ii*) 1.0, (*iii*) 1.5 and (*iv*) 5.0.

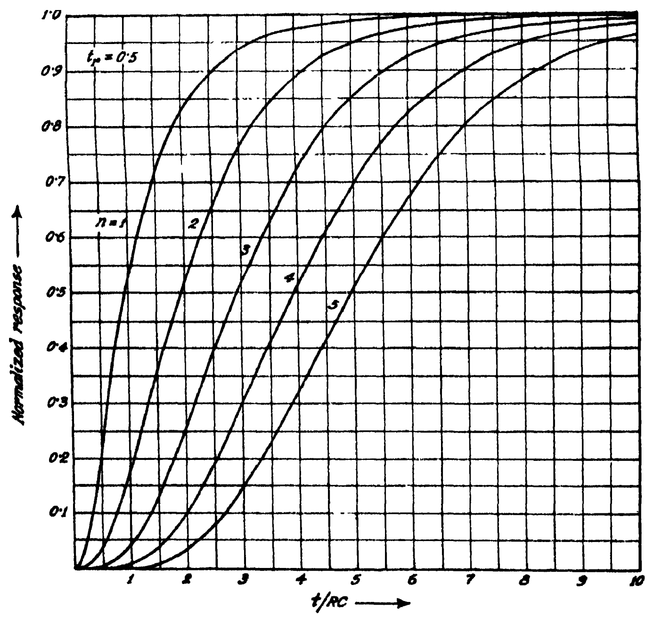

Fig. 8. Response of a *RC*-coupled pulse amplifier to a ramp function with rise time $t_r = 0.5$

That the response to a ramp function with a very sharp rise, *e.g.*, 0.1 in terms of *RC*, is practically identical with that to a step function, is obvious

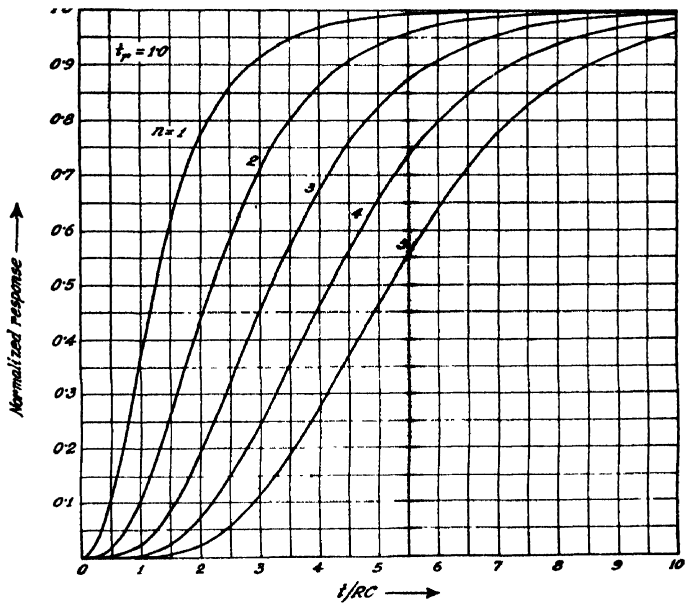

Fig. 9. Response of an *RC*-coupled pulse amplifier to a ramp function with a rise time $t_r = 1.0$.

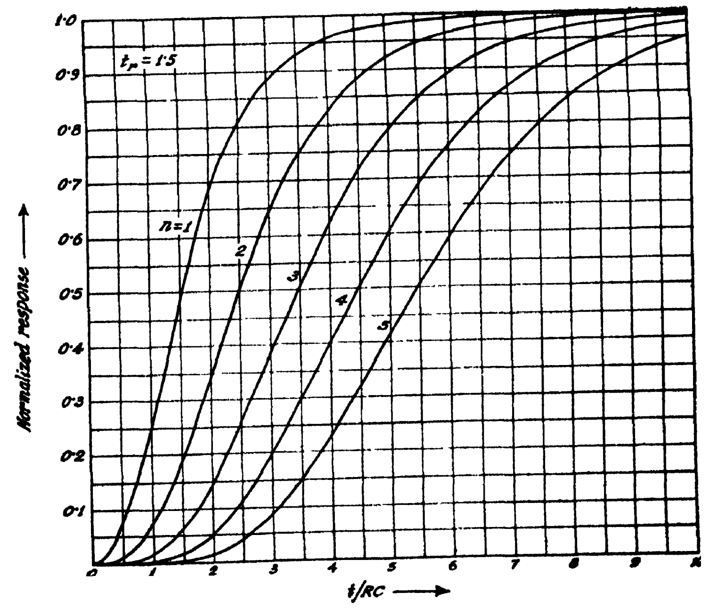

Fig. 10. Response of an *RC*-coupled pulse amplifier to a ramp function with a rise time $t_r = 1.5$.

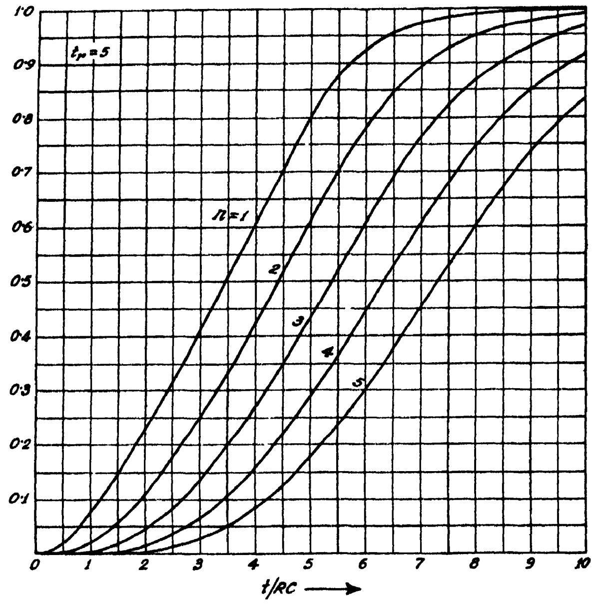

Fig. 11. Response of an RC-coupled pulse amplifier to a ramp function with rise time $t_r = 5.0$.

when we study figuıes 12 and 13. The response functions of RC-coupled pulse amplifiers to a step function input is well known and can be found in standard text-books on pulse technique.

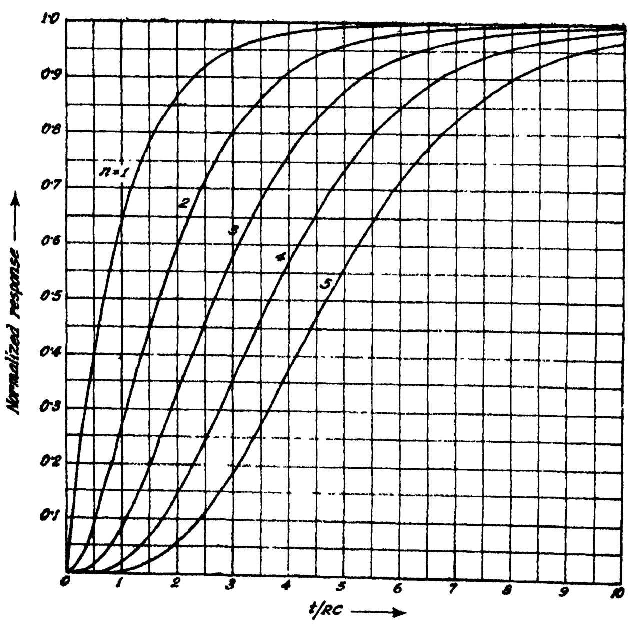

Fig. 12. Response of an RC-coupled pulse amplifier to a step function input.

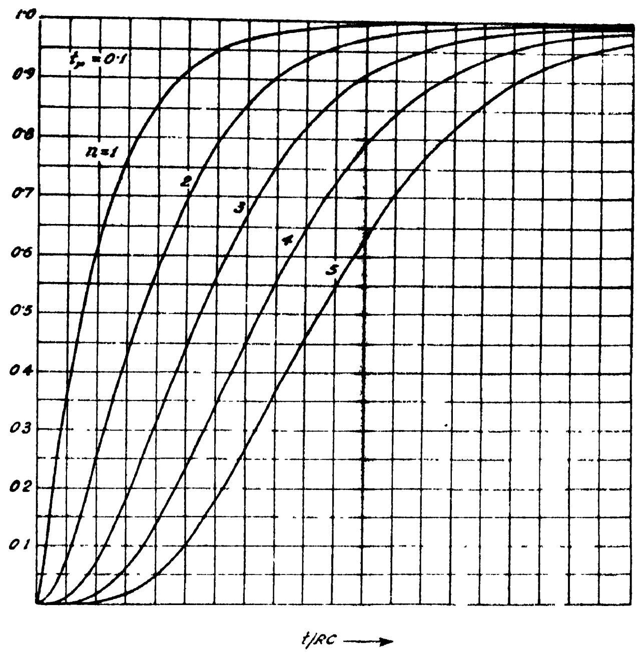

Fig. 13. Response of an RC-coupled pulse amplifier to a ramp function with a rise time t_r = 0.1.

RISE AND DELAY TIMES OF THE OUTPUT PULSE

If we follow the conventional definitions of rise and delay times of the pulse amplifier in this case also, we have :

Rise time $= t_u - t_l = T_r$

Delay time $= t_d$

where t_u, t_l and t_d correspond to the times at which the normalized response assumes the values 0.9, 0.1 and 0.5 respectively The input pulse has finite rise and delay times of its own which are given by the expressions :

10 to 90 per cent. rise times of the input pulse$=0.8t_r$ and delay time$=0.5\ t_r$.

So the actual contribution of the pulse amplifier to the increase of rise time and delay time of the output response is determined by the expressions $(T_r - 0.8\ t_r)$ and $(t_d - 0.5\ t_r)$ respectively.

We shall now derive expressions relating all the three parameters, t_u, t_l and t_d with the rise time t_r of the input pulse for a single stage RC amplifier.

If $t_u \gtrsim t_r$, we have

$$t_u = \log_e \frac{10(\epsilon^{t_r} - 1)}{t_r} \qquad \dots \quad (12)$$

When t_r tends to zero in the limit (step function input),

$$t_u = \log_e 10 = 2.302585$$

When $t_u \leq t_r$, t_u is related to t_r by the expression

$$t_r = \frac{10}{9}(\epsilon^{-t_u} + t_u - 1) \qquad \text{.. (13)}$$

If $t_u = t_r$, we obtain the relation

$$\epsilon^{-t_u} = 1 - 0.1\ t_u \qquad \text{.. (14)}$$

Similarly, t_l is expressed in terms of t_r by the following equations

$$t_r = 10(\epsilon^{-t_l} + t_l - 1),\ t_l \leq t_r \qquad \text{.. (15)}$$

$$t_l = \log_e \frac{10(\epsilon^{t_r} - 1)}{9 t_r},\ t_l \geq t_r \qquad \text{.. (16)}$$

In the case of step-function response $(t_r = 0)$, we have from (16),

$$\underset{t_r \to 0}{\mathrm{Lt}} [t_l] = 0.105360$$

When $t_l = t_r$,

$$\epsilon^{-t_l} = 1 - 0.9\ t_l \qquad \text{... (17)}$$

Solving (17) numerically, we obtain the corresponding value of t_l to be 0.214559. In figure 14 we have plotted t_u, t_l and T_r against rise time t_r of the input pulse. In the same figure we have drawn the curve $(T_r - 0.8\ t_r)$ versus t_r to show clearly the contribution of the pulse amplifier to the increase of rise time of the reproduced pulse. It is found that the increase of rise time is significant at small values of t_r. When $t_r \geq 9.0$, $(T_r - 0.8\ t_r)$ is less than 0.2. This shows that with the decrease of steepness of pulse-fronts the faithfulness of amplification improves to a large extent.

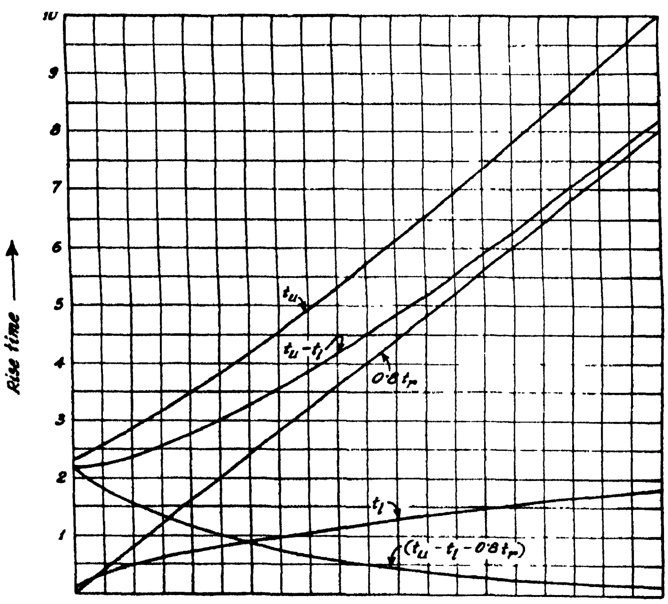

Fig. 14. A plot of rise time of the output pulse as a function of t_r

The delay time t_d is related to the rise time t_r of the input pulse by the expressions:

$$t_r = 2(\epsilon^{-t_r} + t_d - 1), \quad t_d \leq t_r \tag{18}$$

and

$$t_d = \log_e \frac{2(\epsilon^{t_r} - 1)}{t_r} \quad t_d \geq t_r \tag{19}$$

When t_r tends to zero, we can find out t_d with the help of (19).

Thus,

$$\underset{\text{Lt } t_r \to 0}{[t_d]} = \log_e 2 = 0.693147 \quad \ldots \quad (20)$$

which is the expression of the delay time in the case of step function input. When $t_d = t_r$, both (18) and (19) lead to the equation

$$\epsilon^{-t_d} = 1 - 0.5\, t_d \quad \ldots \quad (21)$$

Solving (21) numerically we obtain this particular value of t_d to be

$$t_d = 1.593624 = t_r$$

A plot of delay time t_d versus t_r is given in figure 15. In the same figure the curve of $(t_d - 0.5\, t_r)$ against t_r is also drawn. The delay of reproduction caused by the pulse amplifier is practically constant and nearly equal to one in terms of RC when $t_r > 5$. At smaller values of t_r the delay curve slightly drops to assume the limiting value 0.6931 in the case of step-function response ($t_r = 0$). So we can conclude that while the steepness of pulse fronts decreases the delay introduced by the amplifier does not increase appreciably. It increases from its value 0.6931 at $t_r = 0$ (step-function input) to 0.910203 at $t_r = 3$ and increases slowly to approach unity asymptotically as t_r increases beyond this value.

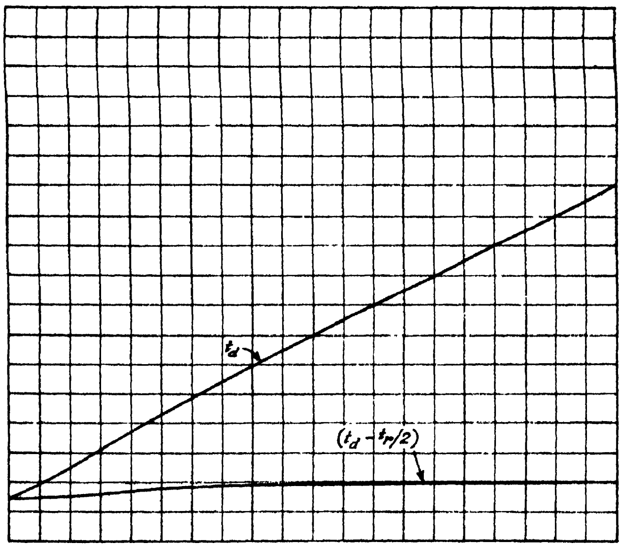

Fig. 15. A plot of delay time of the output pulse as a function of t_r.

RESPONSE TO A PULSE WITH LINEAR RISE AND FALL

It will be assumed that for the purpose of finding out the response it is sufficient to consider only the high frequency equivalent circuit of the amplifier (figure 6).

The output voltage $e_0(p)$ is given by the expression

$$e_0(p) = \frac{g_m R}{1+pCR}\left[\frac{1}{p^2 t_1} - \frac{t_2}{t_1(t_2-t_1)}\cdot\frac{\epsilon^{-pt_1}}{p^2} + \frac{1}{(t_2-t_1)}\cdot\frac{\epsilon^{-pt_2}}{p^2}\right] \quad \ldots \quad (22)$$

If we normalize (22) by substituting $t = t/RC$ and $e_0(t) = e_0(t)/g_m R$, we have

$$e_0(p) = \frac{1}{(1+p)}\left[\frac{1}{p^2 t_r} - \frac{t_f}{t_r(t_f-t_r)}\cdot\frac{\epsilon^{-pt_r}}{p^2} + \frac{1}{(t_f-t_r)}\cdot\frac{\epsilon^{-pt_f}}{p^2}\right] \quad \ldots \quad (23)$$

where $t_r = t_1/RC$ and $t_f = t_2/RC$.

Taking the inverse Laplace transform of (23), we obtain

$$e_0(t) = \frac{\epsilon^{-t}+(t-1)}{t_r}, \quad 0 \le t \le t_r \quad \ldots \quad (24)$$

$$e_0(t) = \frac{\epsilon^{-t}}{t_r}\left[1 - \frac{t_f}{t_f-t_r}\cdot\epsilon^{t_r}\right] - \frac{t-t_f-1}{t_f-t_r}, \quad t_r \le t \le t_f \quad \ldots \quad (25)$$

and

$$e_0(t) = \frac{\epsilon^{-t}}{t_r}\left[1 - \frac{t_f\epsilon^{t_r} - t_r\epsilon^{t_f}}{(t_f-t_r)}\right], \quad t \ge t_f \quad \ldots \quad (26)$$

The nature of the response is illustrated in figures 16-20, where the effect of variation of t_r and t_f is shown clearly.

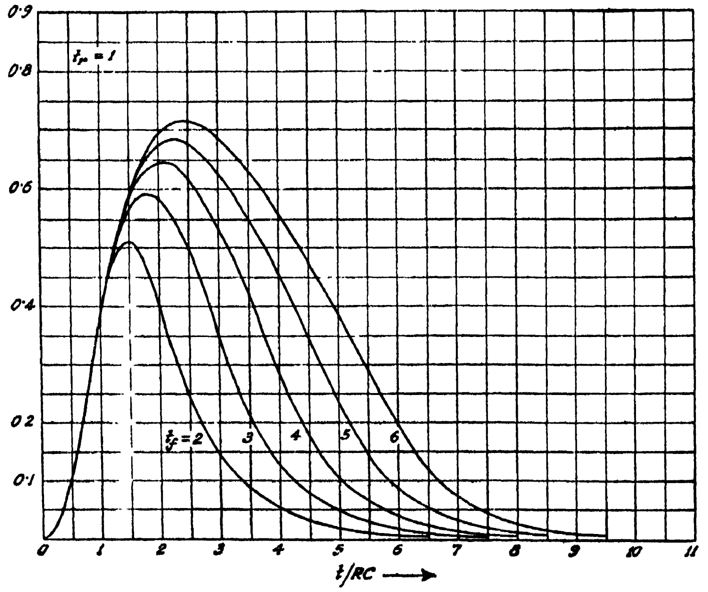

Fig. 16. Response of an RC-coupled pulse amplifier to a pulse with linear rise and fall (rise time $= RC$).

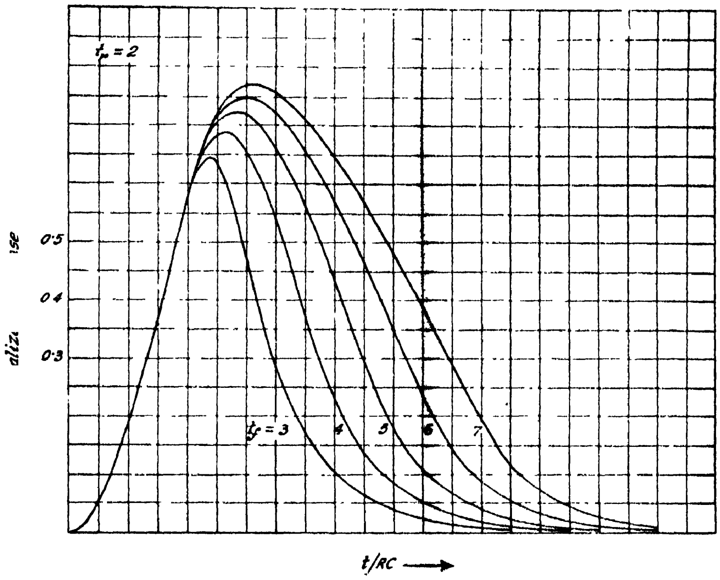

Fig. 17. Response of an RC-coupled pulse amplifier to a pulse with linear rise and fall (rise time $= 2RC$).

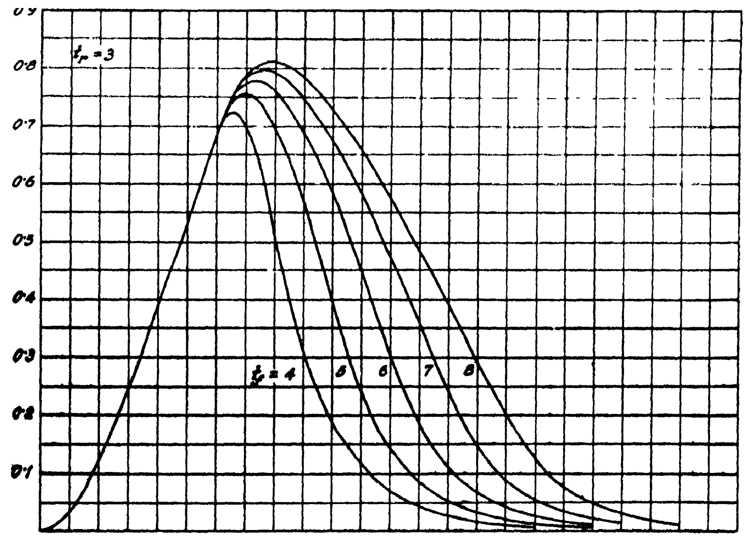

Fig. 18. Response of an RC-coupled pulse amplifier to a pulse with linear rise and fall (rise time $= 3RC$).

It is interesting to note that the maximum amplitude gradually increases as the steepness of fall is reduced for a constant time of rise. The maximum amplitude of the output voltage occurs at a time which lies in the interval $t_r \leq t_m \leq t_f$. So t_m can be determined by solving the equation obtained by

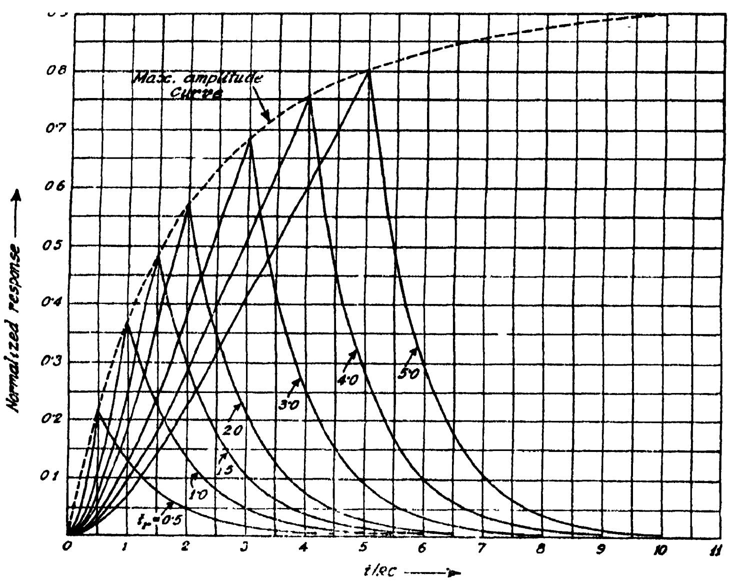

Fig. 22. Response of an RC-coupled pulse amplifier to a saw-tooth pulse with variable rise time. The maximum amplitude versus t_r curve is shown in dotted lines.

It is evident from the plot that the maximum output occurs at $t=t_r$. The maximum amplitude curve as a function of t_r is given by the equation

$$e_0(t_r) = \frac{\epsilon^{-t_r} + (t_r - 1)}{t_r} \tag{32}$$

The maximum amplitude curve is shown in dotted line in figure 22.

CONCLUSION

The object of this paper is to show the changes of response characteristics with the variation of steepness of pulse fronts. It has been shown that a resistance-coupled pulse amplifier does not increase the rise time of the reproduced pulse appreciably if the input pulse has a rise time much greater than RC (*e.g.*, $t_1=9RC$). It has a major contribution only if the steepness of the input pulse front is very great (*e.g.*, $t_1=0.2$ or 0.5 RC). The additional delay introduced by the amplifier to reproduce the input pulse has a value practically equal to RC when $t_1 > 5RC$. It has a minimum value in the case of a step function input pulse. Figure 15 will furnish a ready information about the exact delay time corresponding to a specified t_r.

In the case of pulses with linear rise and fall, it has been noted that the maximum amplitude of the output pulse and the time which corresponds to this maximum, are directly related to times of both rise and fall.

Though the nature of the responses in every case may be drawn from physical principles, a mathematical treatment of the response characteristics of pulse amplifiers to various types of input is necessary for accurate assumptions regarding the design and performance of amplifiers. In this paper we have only discussed about the responses of a simple resistance-coupled amplifier. In a future communication we shall make an attempt to give a detailed analysis regarding the effects of negative feed-back and shunt-compensation on the response characteristics.

ACKNOWLEDGMENTS

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